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RELIABILITY STUDIES OF FINITE ELEMENT METHODS IN NORTH AMERICA (U) ADVISORY GROUP FOR AEROSPACE RESEARCH AND DEVELOPMENT NEUILLY.. J J KACPRZYNSKI JUL 87

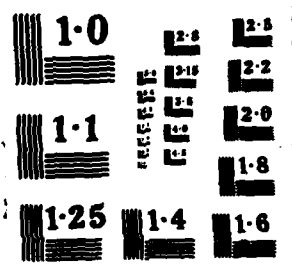
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NORTH ATLANTIC TREATY ORGANIZATION
ADVISORY GROUP FOR AEROSPACE RESEARCH AND DEVELOPMENT
(ORGANISATION DU TRAITE DE L'ATLANTIQUE NORD)

AGARD Report No.748
**RELIABILITY STUDIES OF FINITE ELEMENT METHODS
IN NORTH AMERICA**

by
Jerzy J.Kacprzyński

Paper presented at the 62nd Meeting of the Structures and Materials Panel of AGARD, Høvik (Oslo),
Norway, 13—18 April 1986.

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Published July 1987

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ISBN 92-835-0423-2



Printed by Specialised Printing Services Limited
40 Chigwell Lane, Loughton, Essex IG10 3TZ

PREFACE

This report discusses the reliability of finite element analysis techniques in place of structural test in the context of airworthiness assessment. Using an example as illustration, the paper shows that in order to obtain meaningful results, the analysis must be performed with extreme care by an experienced analyst. The reliance on the finite element analysis alone may result in serious misdiagnosis. The verification and the certification of programmes and the certification of users are discussed. Some of the most important tests for validation of finite element programmes are presented.

Ce rapport traite de la fiabilité des techniques d'analyse par la méthode des éléments finis utilisées à la place d'un essai structural dans le contexte de l'évaluation de la navigabilité aérienne. Illustrant sa démonstration par un exemple, l'auteur montre que, pour fournir des résultats significatifs, l'analyse doit être effectuée avec le plus grand soin par un analyste expérimenté. Le fait de se fier uniquement à l'analyse par la méthode des éléments finis peut donner lieu à une grave erreur de diagnostic. La vérification et la certification des programmes ainsi que la certification des utilisateurs sont prises en considération dans le rapport. Quelques uns des essais les plus importants visant à la validation des programmes d'éléments finis sont décrits.



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RELIABILITY STUDIES OF FINITE ELEMENT METHODS IN NORTH AMERICA

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SUMMARY

As a result of increasing costs of aircraft structural test, the use of finite element analysis in aircraft certification has been proposed many times. The replacement of test by analysis can be done safely only when the reliability of finite element analysis is proved. Using an example as illustration, the paper shows that in order to obtain the meaningful results, the analysis must be performed with extreme care by an experienced analyst. The reliance on the finite element analysis alone may result in serious misdiagnosis. The verification and the certification of programmes and the certification of users are discussed. Some of the most important test for validation of finite element programmes are presented.

INTRODUCTION

The structural tests (static, dynamic and fatigue) on all new aircraft to validate the design strength and the analytical predictions for stress, deformation and life are growing more complex, more costly and time consuming. At the same time the development of finite element analysis techniques allied to the increasing power and reducing cost of computers is leading to the position where analysis may replace at least some of the tests required for the qualification of structural integrity. If computer based finite element analysis is to take a more dominant position in the proof of structural integrity, consideration must be given to the qualification of the process of analysis, namely to the qualifications of the finite element programmes, analysis and the modelling.

THE COMPUTER SIMULATION OF EXPERIMENTAL TESTS

The replacement of tests by computer simulation may be justified only when the analysis produces the unique and the correct results, which for the time being not always can be achieved with finite element analysis. The problem of accuracy of finite element has been studied extensively for many years. To illustrate the problem let us consider one of the cases discussed on the Second International Conference on Computational Methods and Experimental Measurements (Ref. 1), namely let us consider a problem of a cylindrical pressure vessel presented by C.G. Floyd from Lloyd's Register of Shipping (Ref. 1, pp. 6-73 to 6-77). The vessel is shown in Figure 1 and is loaded with internal pressure $p = 2.61$ psi.

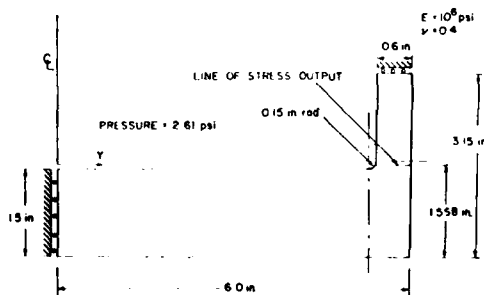


Figure 1. Pressure vessel problem

This study arose from an analysis of the cause of surface cracking at the internal corner. Two initial finite element analyses gave differing results and an independent validation check was therefore necessary. The validation was carried out by Floyd using an idealised pressure model which was analysed both by numerical techniques and by an experimental photoelastic study. The photoelastic model was constructed from epoxy resin, which has a Poisson's ratio of 0.5. The area of interest is the corner fillet, for which the peak surface stress around the corner was required, together with the variation of the stresses through the cylinder wall at the location of the peak stress. The line for which results

were obtained from the photoelastic model is shown on Figure 1. This line passes through the point of maximum surface stress around the corner.

The finite element analyses were performed with NASTRAN, ADINA, ANSYS, ABAQUS, FEMEP and PAPEC, but only the results of NASTRAN and ADINA were shown in Reference 1. The NASTRAN finite element model was constructed from axisymmetric plane strain trapezoidal and triangular elements with a total of twelve triangular elements covering the surface of the corner. The material properties used were those of epoxy resin, although a Poisson's ratio of 0.4 was assumed as the programme would become unstable for a Poisson's ratio of 0.5. A range of Poisson's ratios between 0.4 and 0.49 was tested, by Floyd, but it was found that for all values above 0.4 instabilities and overestimations were occurring.

The ADINA model used by Floyd was constructed with axisymmetric eight noded isoparametric quadrilateral elements, and a total of six elements covered the 90 degrees around the corner. A coarser mesh than that used for the NASTRAN model was considered justifiable in view of the higher order elements used. The ADINA incompressible material capability could not be used as it is not applicable to axisymmetric elements, and so a simple linear elastic analysis was performed with a Poisson's ratio of 0.4. For higher values of Poisson's ratios instabilities and inaccuracies occurred.

The boundary element analysis was performed with BEASY programme using axisymmetric three noded isoparametric line elements with the inner corner surface defined by four elements. In the boundary element method the instability problem is less severe than with finite element method and therefore a Poisson's ratio of 0.49 could be used.

The plots of major principal stress along the line of interest calculated with finite element programmes ADINA and NASTRAN, with boundary element programme BEASY and from photoelastic model are shown in Figure 2.

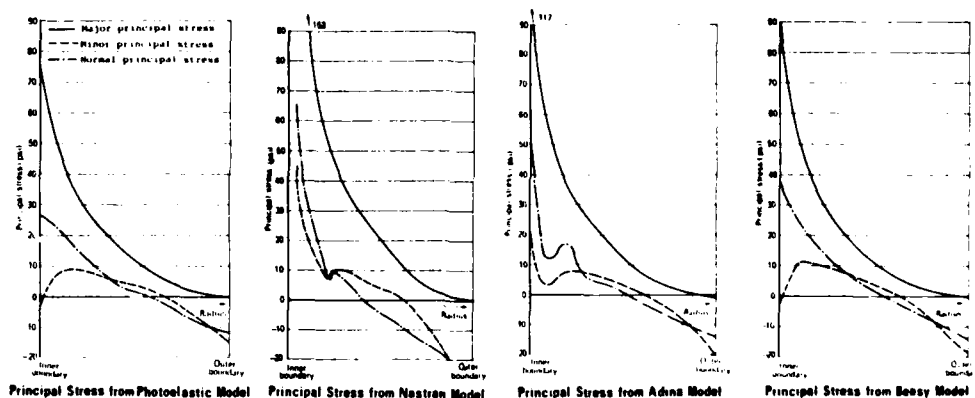


Figure 2. Floyd's experimental and numerical results for vessel problem (Ref. 1)

Although the general variation of the principal stresses through the cylinder wall is similar, the magnitude of the peak stress on the inner surface varies considerably. The photoelastic model gave the lowest value of 76 psi and the finite element models produced 117 and 153 psi for ADINA and NASTRAN models respectively. The results of ANSYS, ABAQUS, FEMEP and PAPEC finite element analyses similarly overestimated the local stress. Floyd indicates that this example represents a type of engineering problem for which the finite element method has been hailed as the ideal solution, and warns that reliance on the finite element analysis alone could have resulted in serious misdiagnosis.

The second study of the Floyd's vessel problem was done by the developer of ADINA programme, K.J. Bathe and T. Sussman. They presented the results of their extensive ADINA analysis on the 5-th ADINA conference (Cambridge, Mass. June, 1985). Some of their results are printed in Reference 2. The vessel was studied with several idealization, starting from 69 elements up to 701 elements. The authors discussed the difficulties connected with obtaining the correct results in ADINA analysis of the vessel problem. Some of their results for 491 element idealization are shown in Figure 3. The paper shows that - contrary to Floyd's conclusion - the finite element method, when correctly applied, can be used in a routine manner to accurately compute the stresses in the pressure vessel. The 69 eight-node element mesh used in the initial analysis gave large stress jumps between elements and in particular near the inner surface of our pressure vessel. The major principal stress on the inside surface was overestimated. The uniformly refined mesh of 491 eight-node element, was overrefined in the region far from the fillet. Finally the manually refined in the fillet region the 69 element mesh up to total of 240 element gave the results comparing favorably with the experimental and boundary element results given by Floyd. The value of the major principal stress on the inner surface at the line of interest was 96 psi. Bathe indicated during the Cambridge meeting that one of the future version of ADINA, to be realised in four, five years will simplify the present difficulties, namely the programme will analyse the discontinuities in stresses in elements, check their equilibrium, and in adoptive way will refine locally the mesh automatically.

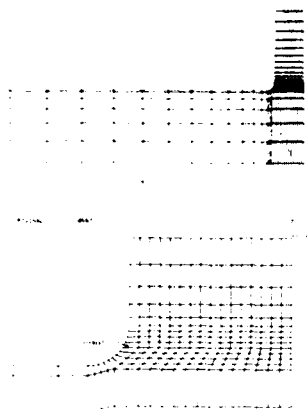


Figure 3. Bathe and Sussman 491 element model of the mesh
(a) complete model
(b) detail of mesh

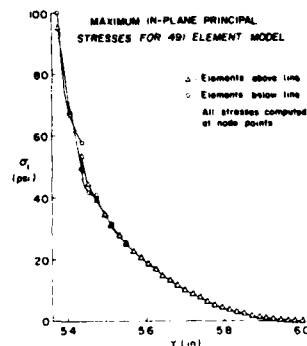


Figure 4. Bathe and Sussman results for 491 element model of the vessel

In response to Floyd's warning against the use of finite elements and Bathe's difficulties in the solution of this problem, the author of this paper performed an independent finite element solution of the vessel problem, using only 25 12-node isoparametric elements and APES programme. The idealization is shown in Figure 5. The contour plots of the maximum principal stress for the whole vessel and for the

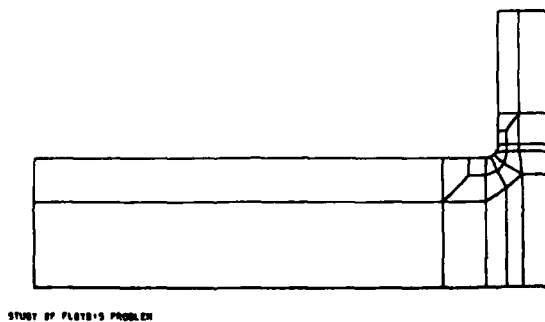


Figure 5. 25 element idealization for APES programme.

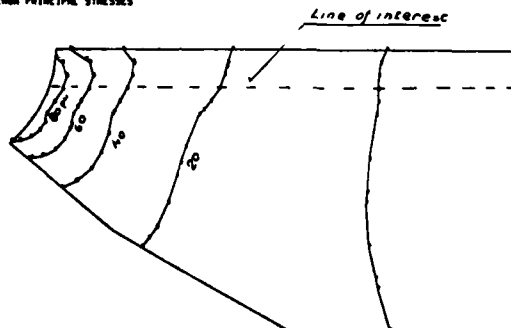
MAXIMUM PRINCIPAL STRESSES



STUDY OF FLOYD'S PROBLEM

Figure 6. Plot of the maximum principal stress with concentration of stress in the corner.

MAXIMUM PRINCIPAL STRESSES



STUDY OF FLOYD'S PROBLEM

Figure 7. Plot of the maximum principal stress in the region of stress concentration

region of stress concentration are shown in Figures 6 and 7. The results agree favorably both with the Floyd's experimental and boundary element results and with the Bathe's ADINA 240 and 491 element idealization. The value of the major principal stress on the inner surface near the line of interest was 88 psi. The results were obtained without any difficulty.

The above discussed pressure vessel problem illustrates well the present state of the numerical analysis of structures. Three experts solving this simple engineering problem, arrive at three different results and conclusions. Floyd's does not recommend to use finite elements, Bathe produces the correct results but with large effort and the present author obtains the solution without any problem. A question is how average stress analyst, generally less experienced and having no experimental data for verification of his results, can produce the correct results?

One can say that the vessel pressure problem is not representing a typical airframe problem, because it is a stress concentration problem. But any airframe has numerous regions of the stress concentrations and usually in these regions the failure of the structure initiates. The correct prediction of the local stresses is extremely important.

Why the above experts arrived at different conclusions? Simply the finite element analysis is not the exact one and does not produce the unique solution. The results are approximate at the best, but very often they are completely wrong because:

a. many elements behave poorly in off-design conditions.

The elements are usually designed for some idealized conditions. For example - the plate element is usually designed in rectangular shape. Very often when such an element is used in trapezoidal form, the results are inaccurate. Sometimes, for the certain load condition the element becomes artificially stiff (a locking phenomenon) or behave, like a mechanism. Sometimes the accuracy of the location of the midsize nodes is extremely important.

b. poor finite element model.

The model may be too coarse, it may have wrong elements. It may be based on a set of assumptions which seem reasonable, and may produce results consistent with these assumptions, but which are incorrect.

c. the limitation of the programme used.

There are assumptions implicit in the mathematical theory and limitations built into the programme code. There are also undetected bugs in the programme.

d. numerical inaccuracy of the analysis.

The combining effect of rounding errors, modelling approximations and other errors leads to a typical accuracy that cannot be guaranteed to be better than 10 per cent under normal conditions.

e. improper use of the code.

The programme is usually used as a "black box" - not adequate and not clear documentation very often leads to improper use of the code.

f. poorly qualified user.

Very often the code users extend themselves beyond their range of expertise.

Very often it is assumed that when the computer prints the output, then the results must be correct. Unfortunately seldom it is true. At present the finite element analysis is not fully reliable and must be performed with care and by qualified people - in structural mechanics and in the computer analysis.

No numerically calculated solution should be accepted without some degree of questioning.

NORTH AMERICAN RELIABILITY STUDIES OF THE FINITE ELEMENT ANALYSIS

In the first decade of the finite element application, namely between the mid fifties and the mid sixties, there were no general, commercially available finite element programmes - anybody, who wanted to perform a finite element analysis, had to develop his own code. In this situation the comparison and the verification of results obtained by different computing groups was rather difficult. It became obvious that some kind of standardization was necessary. As a result of this NASA financed a development of NASTRAN, which was supposed to become a standard programme for all aeronautical analyses. Unfortunately, it never happened and there are mainly three reasons for this.

The first reason is that there was not enough money assigned for the development of this code - initially it supposed to be both the displacement and the force programme, but only the displacement code was developed.

The second reason was insufficient support for this programme - instead of giving the proper technical support, NASA allowed some of the original developers of NASTRAN, to create their own company and to develop further the code. In this way the MacNeal-Schwendler NASTRAN was created and instead of one standard programme, there were two programmes, NASA owned COSMIC-NASTRAN and privately owned MSC-NASTRAN developed by MacNeal-Schwendler Corporation.

The third reason was caused by universities - namely universities teaching finite elements methods do not teach how to perform the analysis correctly, but teach how to develop new finite elements and how to write programmes. Many fresh graduates, noticing the problems with the use of existing programmes receive the permission to develop the new ones and repeat the old mistakes. As a result of this we have on the market many finite element programmes, some quite good, but none a perfect one, and none a standard one.

The issue of verification and qualification of computer software has been raised many times in the last two decades. A distinction has to be made between verification and qualification. Verification is defined as the demonstration that a computer programme does correctly what it is supposed to do, i.e., it solves correctly the model that was programmed, independently of whether or not the model is a valid representation of any particular system. Qualification is more concerned with the use of computer programmes for solving specific problems encountered in design. Given that a computer programme does what it is supposed to do, does the combination of mathematical model, description of material properties representation of loading system, and boundary conditions combine to give an accurate representation of the physical system? Several methods of qualification are described by Griffin [3]. The concept of bench mark calculations was introduced by Wrigth [4] as a means of verification and qualification. The idea is to establish a set of problems with known solutions which can be used for comparison. If a computer programme solves all the bench mark problems correctly, it is then considered to be verified. The problem is that exact solutions, and even experimental results, are available only for limited classes of problems. Thus, it is impossible to verify all programme options of interest by this means.

Certification of computer programmes, for use in design has been proposed by Rashid [5]. He outlined a procedure which would lead to evaluation of each programme by a committee that would approve or disapprove the programme for use. Evaluation of the programme would not only be concerned with the validity of the programme but also with the defensibility of the theory and the adequacy of documentation.

Responsibility for providing the committee with material necessary for evaluation would rest with the programme developer. Rashid admits there are arguments against certification - legal ramifications, magnitude of test, violation of property rights, etc - but disposes of them and concludes that certification is necessary if highly complicated computer programmes are to be used in design analysis by people who cannot possibly investigate the programmes in sufficient detail to satisfy themselves as to their validity.

O'Donnell [6] points out that a certification requirement would stifle development of new computer programmes, provide little assurance that the programmes would be applied properly to a given design problem, and would require certification of every version of a programme on every different computer. As an alternative he proposes an independent review of stress reports. Many suggested that cross-checking of results by independent analyses is the best assurance of reliable results from advanced computer programmes.

All of the concepts discussed so far, with the exception of cross-checking of results, are concerned with verification, qualification and certification of the computer software. However, as pointed out by Griffin [7], there is no such thing as absolute verification. No matter how meticulous the programmer, all computer programmes contain errors, and they can only be found by use of the programme on a broad range of real problems. The user must be alert to the possibility of errors and check his results thoroughly. The qualification is the responsibility of the user. He must select the proper programme, inputs, and control parameters to give accurate results for his particular problem. The user must qualify a programme for use in each new design situation. Bench mark calculations are very useful, and assist the developer and user in verification and qualification, but alone are not enough. They are a necessary condition for verification but not a sufficient condition.

There was a lot of discussion about the verification and certification of the programmes and the certification of the users but nothing was actually done. There is no organisation in North America having the authority to do something about this important problem. Finally after the collapse of the roof of Civic Centre arena in Connecticut caused by the improper use of the code in the finite element analysis (Ref. 8 and 9), the group of the concerned experts decided to do something about it. Using the American Institute of Aeronautics and Astronautics as a basis of the activities, this group developed the voluntary standards for finite element analysis. The standards cover only the linear static analysis.

The developers of the major commercial finite element programmes have been involved in this activity - they represent ABAQUS, ADINA, ANSYS, ANSWERS, SRAC/COSMOS7, EISI/EAL, EASE, SSD/FACTS, FLUSTR, SDRC/FRAME, GIFTS, GNATS, MARC, MSC/NASTRAN, UAI/NASTRAN, NFAP, PDA/PATCHES III, PDA/PATRAN, SAP80, SPAR, SHORE, SUPERB, SUPERTAB.

In the public discussion on two AIAA Structures, Structural Dynamics and Material Conferences, the set of test cases with known solutions has been proposed. These test cases can be used for voluntary verification of codes and elements.

PROPOSED FINITE ELEMENT STANDARD TEST PROBLEMS

The selected bench mark test cases have known solutions which allows for verification of the finite element programmes. The verification is voluntary - there are no organization having an authority to enforce it. The selected cases represent the most important parts of the static analysis. The most important tests represent the following cases:

basic patch tests

beam tests

- straight beam
- curved beam
- twisted beam

plate tests

- single curved shell

- double curved shell

- axisymmetric solid

- 3D solid

thermal problems

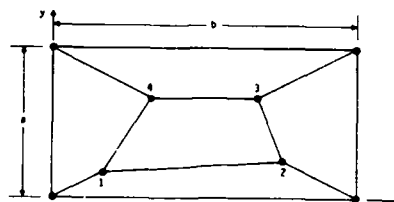
Each test case may be studied with several variations of:

- mesh - regular, irregular, location of midside nodes
- loads - mechanical, gravity, centrifugal, thermal
- output - displacements, stresses, reactions
- elements - quadrilaterals, triangles, with midside nodes, other
- materials - isotropic, variable Poisson's ratio, plane strain
- numbering - with reordering of the element grid data.

Patch tests

The basic test which all elements should pass is a developed by Irons [10] patch test. Most of the elements are designed in a form of some idealized regular shape - for example a plate element is usually designed in a rectangular shape. When such an element is used in other shape, often the accuracy of the element significantly deteriorates.

The patch test has to demonstrate that the element having an irregular shape reproduces the uniform displacement or stress distribution as specified on the boundary of the patch. Two patch tests have been specified - one for plates [Fig. 8] and one for solids [Fig. 9].



$$a = .12; b = .24; t = .001$$

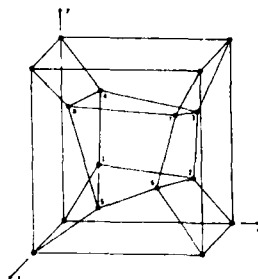
$$E = 1.0 \times 10^6; \nu = 0.25$$

Boundary conditions: see Table 1

Location of inner nodes:

	x	y
1	.04	.02
2	.18	.03
3	.16	.08
4	.08	.08

Figure 8. Patch test for plates.



Outer dimensions: unit cube

$$E = 1.0 \times 10^6; \nu = .25$$

Boundary conditions: see Table 1

Location of inner nodes:

	x	y	z
1	.249	.242	.192
2	.826	.288	.288
3	.850	.609	.263
4	.273	.750	.230
5	.320	.186	.643
6	.677	.305	.683
7	.788	.693	.644
8	.165	.745	.702

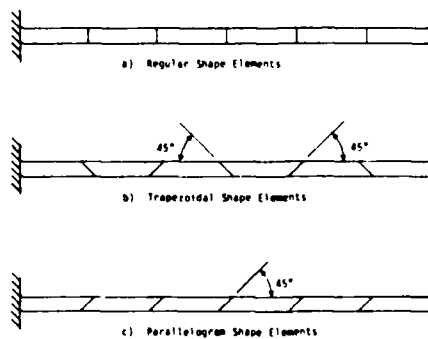
Figure 9. Patch test for solids

Irons suggested that the elements not passing the patch test should be rejected. From the preliminary verification of the elements used in the commercially available finite element programmes it appears that most of the used elements do not pass these tests. The elements which pass the patch tests usually are more complex and increase the cost of analysis - therefore the developers prefer to use the simpler elements. The reduction of the cost of analysis with the increased risk of producing incorrect results should be avoided.

Beam tests

Three tests for beam elements are proposed.

- straight cantilever beam modelled with rectangular elements, trapezoidal shape elements and parallelogram shape elements (Fig. 10)
- curved beam (Fig. 11)
- twisted beam (Fig. 12)



Length = 6.0; width = 0.2
Depth = 0.1

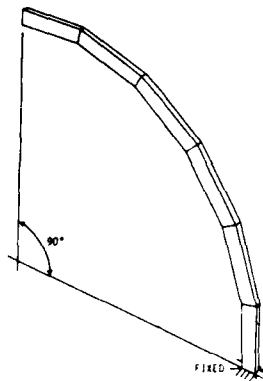
$$E = 1.0 \times 10^7; \nu = 0.30;$$

Mesh = 6x1

Loading: unit forces at free end

Note: All elements have equal volume

Figure 10. Straight cantilever beam.



Inner radius = 4.12
 Outer radius = 4.32
 Arc = 90°
 Thickness = 0.1
 $E = 1.0 \times 10^7$; $\nu = 0.25$
 Mesh = 6x1
 Loading: Unit force at tip

Figure 11. Curved beam



Length = 12.0; Width = 1.1,
 Depth = .32
 Twist = 90° (root to tip)
 $E = 29.0 \times 10^6$; $\nu = 0.22$
 Mesh = 12x2
 Loading: Unit forces at tip

Figure 12. Twisted beam

Plate tests

Plate elements are tested in a rectangular form plate as shown in Figure 13 or in a segment of a ring (Fig. 14) simply supported at two sides and loaded with concentrated normal forces applied at one of the points A, B and C.

Shell elements

Shell elements can be tested in several ways. A basic test is in single curvature Scordelis - Lo roof shown in Figure 15. The value for the midside vertical displacement quoted in Reference 13 is 0.3086.

The test of a membrane action without bending is performed in a patch test for a cylindrical shell under internal pressure (Fig. 16). For simplicity one may consider only hoop pressure without axial loading and set Poisson's ratio $\nu = 0$. Then a patch test can be made using four elements with prescribed displacements at the eight boundary nodes and a radial nodal load Q_c at the center node c. To pass the patch test w_c should be equal to pr^2/Et . The test is to find out how small should the angle ϕ be in order to obtain satisfactory results. It is also worthwhile to test irregular shaped elements (Ref. 14)

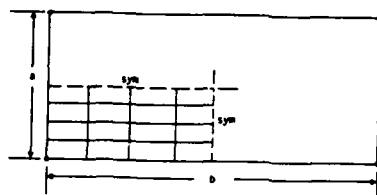
For torsion of a split cylindrical shell the only significantly contribution to the stress field is a constant torsional couple $M_{x\phi}$ and the displacement field has no component normal to the surface. To test stress couple action under in extensional condition one may consider only a simple element $\phi R \times a$. The equivalent nodal loads are nodal forces F_1 and F_2 as shown in Figure 17 with

$$F_2 = F_1 \cot \frac{1}{2} \phi \quad \text{and} \quad F_1 = \frac{T\phi}{4R}$$

the exact solution is

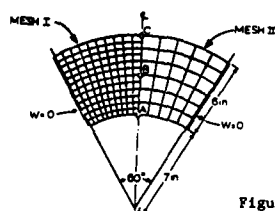
$$M_{x\phi} = \frac{T}{2\pi R}$$

$$\text{rate of twist } \theta = 3T/2\pi R h^3 G$$



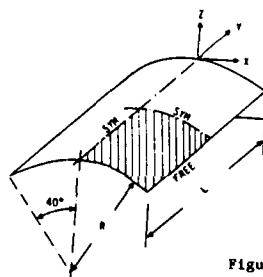
$a = 2.0$; $b = 2.0$ or 10.0 ;
 thickness = .0001 (plates)
 thickness = .01 (solids)
 $E = 1.7472 \times 10^7$; $\nu = 0.3$
 Boundaries = simply supported or clamped
 Mesh = $N \times N$ (on t of plate)
 Loading: Uniform pressure $q = 10^{-4}$ or
 central load $p = 4.0 \times 10^{-4}$

Figure 13. Rectangular Plate



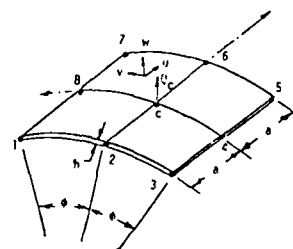
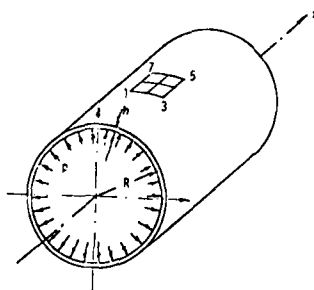
$E = 4.6 \times 10^5$ lb/in²; $\nu = 0.35$
 thickness = 0.168 in
 unit loads at one of points A, B, C
 solution in Ref. 11, 12

Figure 14. A curved simply supported plate



Radius = 25.0; length = 50.0;
 thickness = 0.25
 $E = 4.32 \times 10^6$; $\nu = 0.0$;
 Loading = 90.0 per unit area in -Z direction
 $u_x = u_z = 0$ on curved edges
 Mesh: $N \times N$ on shaded area

Figure 15. Scordelis-Lo roof



$w_{,n}$ and $w_{,s}$ at node 1 to 8 are zero
 $w_1 = w_2 = w_3 = w_4 = w_5 = w_6 = w_7 = pR^2/(Et)$
 $u_1 = u_2 = u_3 = u_4 = u_5 = u_6 = u_7 = u_8 = 0$
 $v_2 = v_6 = 0$; $v_3 = v_4 = v_5 = -pR^2/(Et)$; $v_1 = v_8 = v_7 = pR^2/(Et)$
 $Q_c = \phi R p$

Figure 16. Membrane action without bending test.

To pass the test not only the solutions for torsional couple $M_{x\theta}$ and rate of twist θ should be accurate but also the membrane stress components N_x , N_θ and $N_{x\theta}$ should be small (Ref. 14)

The element geometric distortion effect is examined in a twisted ribbon idealized by shell elements (Fig. 18) and References 15 and 16.

The pinched cylinder (Fig. 19) with a diaphragm is one of the most severe test for both in extensional bending modes and complex membrane states (Ref. 17). Any element that passes the diaphragm support test will perform well when the boundary condition is simplified to a free boundary.

For double curvature shells two tests are proposed. The hemispherical shell problem (Fig. 20 and Ref. 17) is a challenging test of an element's ability to represent in extensional modes; it exhibits

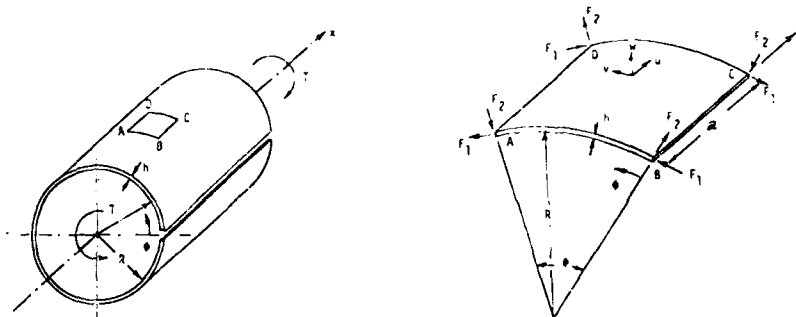
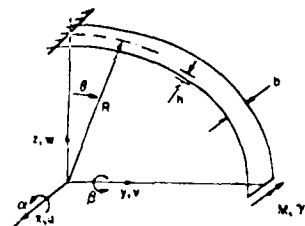


Figure 17. Stress couple action under inextensional condition



$$E = 2.1 \times 10^6$$

$$\nu = 0.3$$

$$h = 1.0$$

$$b = 1.0$$

$$R = 100$$

Analytic solution

$$\gamma = M/R/(2EI)$$

Figure 18. Test of effect of element geometric distortions

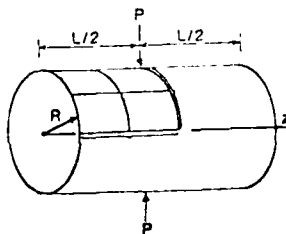


Figure 19. Pinched cylinder test

almost no membrane strains. The role of this test problem is less critical with regard to inextensional bending than the pinched cylinder problem. However, it is very useful problem for checking the ability of the element to handle rigid body rotations about normals to the shell surface. Large sections of this shell rotate almost as rigid bodies in response to this load, so that the ability to accurately model rigid body motion is essential for good performance in this problem. Some 5 degree of freedom per node formulations of triangular elements fail this test because they result in spurious straining when rotated about the normal to the shell surface.

The second hemispherical shell test is shown in Figure 21. The solution of this problem is given in Reference 18 for a slightly different configuration in which the hole at the axis is closed.

The equator is a free edge so that the problem represents a hemisphere with four point loads alternating in sign at 90° intervals on the equator. The hole at the top has been introduced to avoid the

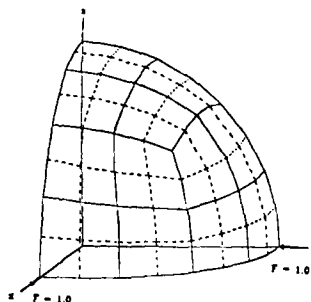
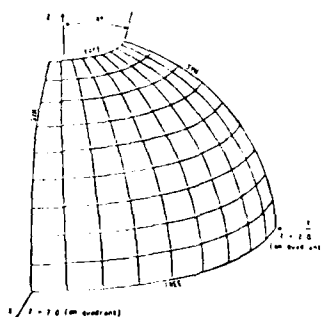


Figure 20. Hemispherical shell problem



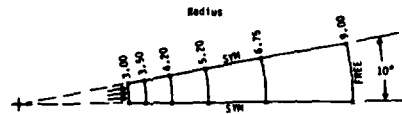
Radius 10.0
 Thickness .04
 $E = 6.825 \times 10^7$
 $\nu = 0.3$
 Mesh = $N \times N$ (on quadrant)
 Loading: concentrated forces as shown

Figure 21. The second hemispherical shell problem

use of triangles near the axis of revolution. Convergence can be studied by varying mesh size. Both membrane and bending strains contribute significantly to the radial displacement at the local point.

Axisymmetric solid

It is proposed to test a 10° segment of the thick-walled cylinder (Fig. 22) under unit internal pressure.



inner radius = 3.0

outer radius = 9.0

thickness = 1.0

E = 1000

ν = 0.49; 0.499; 0.4999

Plane strain conditions

Mesh 5x1 (as shown)

Figure 22. Thick-walled cylinder

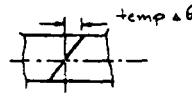
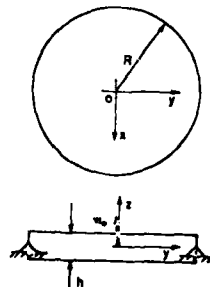
The test is supposed to examine the effect of nearly incompressible material. The assumed plane strain condition along with the radial symmetry, confines the material in all but the radial direction and intensifies the numerical difficulty caused by near incompressibility.

Solid elements

The proposed test of solid elements is in a form of a quarter of a rectangular plate (Fig. 13) modelled with solids with thickness 0.01.

Thermal problems

The proposed tests include two plates and one solid body with temperature gradient. In a case of the simply-supported circular plate (Fig. 23) subjected to a constant through the thickness temperature gradient there is known analytical solution (Ref. 19).



R/h = 10; E; ν; α

Analytical solution: $w_0 = \frac{\alpha \Delta \theta}{h} R^2$

all stresses σ_{ij} are zero

Error indicators: $E_W = \frac{|w_0^{FE} - w_0|}{w_0}$

$E_\sigma = \max \frac{|\sigma_{ii}|}{E \alpha \Delta \theta}$

Figure 23. Thermal problem of a circular plate.

The second plate thermal problem is the free expansion of a rectangular plate with a linear temperature distribution (Fig. 24 and Ref. 20).

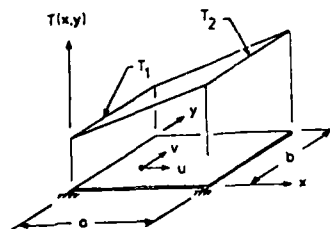


Figure 24. Free expansion of plate under linear temperature

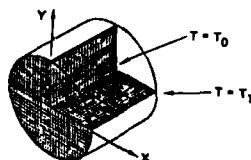
The analytical solutions are:

$$\begin{aligned} T(x,y) &= T_1 + \frac{y}{b} (T_2 - T_1) \\ u(x,y) &= \alpha T_1 + \frac{\alpha}{2b} (x^2 - y^2) (T_2 - T_1) \\ v(x,y) &= \alpha y T_1 + \frac{\alpha}{b} xy (T_2 - T_1) \\ \sigma_x = \sigma_y = \tau_{xy} &= 0 \end{aligned}$$

A single quadrilateral finite element solution should be compared with the analytical solution for displacements and stresses.

Thermal stress analyses are complicated by material geometry in high-temperature composite structure in much the same way that shell geometry complicates stress analyses for metallic structures. In local material coordinates, most composites are orthotropic, but in structural coordinates, they are often anisotropic because of material geometry. Three thermal stress problems are proposed for which there are exact material geometry models and, in one case, an exact thermal stress solution. All three structures are axisymmetric, but material geometry causes two of the three to be anisotropic in cylindrical coordinates. Significantly fiber stress errors occur when the anisotropic cone is approximated as orthotropic. These problems are proposed as verification problems for codes with composite and anisotropic capabilities.

The first problem is the Carterian orthotropic disk problem shown in Figure 25 and having the analytical solution (Ref. 21).



Material geometry
 $\alpha = \gamma = \phi = 0$

Disc geometry
 $R = 1$ unit

Material properties (non-dimensional)

$$E_x/E_y = 1 \quad E_x/E_z = 1$$

$$E_x/G_{xy} = 17.25 \quad \nu_{xy} = 0.05$$

$$\alpha_x/\alpha_y = \alpha_x/\alpha_z = 1$$

$$\beta = 0.0820734341$$

Figure 25. Carterian orthotropic disk problem

The other two problems are the bidirectional tape-wrapped cone (Fig. 26) and the bidirectional laminated involute cone (Fig. 27). The involute constants are defined in References 22, 23.

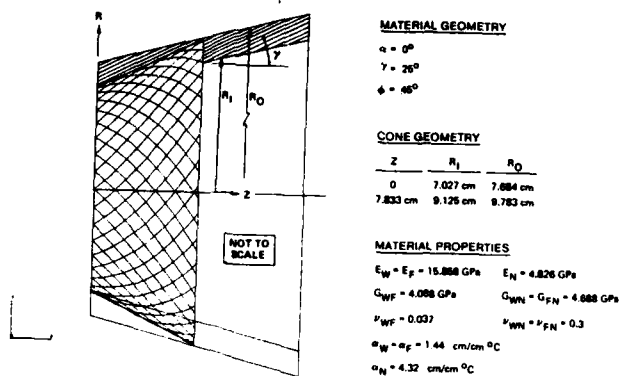


Figure 26. Bidirectional tape-wrapped cone

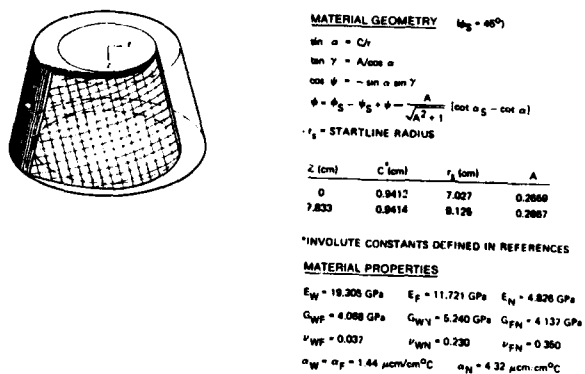


Figure 27. Bidirectional laminated involute cone

FINITE ELEMENT USER COMPETENCY ISSUE

As the design of sophisticated aircraft becomes more dependent upon complicated computer programmes, it becomes apparent that solution accuracy becomes more dependent upon the skill and knowledge of the user. The competency issue of the finite element user has been debated since late 1960s. Nickell [24] proposed that people as well as programmes be qualified. He outlined an instructional programme balanced between the fundamentals of the physical theory and practical application of computer programmes on design problems. Modern teaching aids, such as video tape and time-sharing computer facilities, would enable the programme to have wide applicability. Griffin [7] has proposed formal certification of users in a way similar to the Professional Engineer registration programme, which requires demonstration of basic understanding of the numerical methods and computer programmes being used. Yates [25] and Rosen [26] also emphasize the need for qualification of users. Both feel strongly that the quality of results is highly dependent upon the user and that he alone is responsible for their validity. Yates states that formal verification and qualification requirements should be imposed on a user and company basis rather than on programme basis. Many other agree that the need for Professional Licensing Agencies to certify engineers is greater than the need to certify programmes.

The user of computer software is defined as the individual who applies computer programmes in the analysis and design of structural components. The developer is the individual who develops programmes for use in analysis. The developer may also be a user, in which case he is intimately familiar with the programme and its use. However, we are primarily concerned with the user, who is not the developer, i.e., the user who is using computer software developed by someone else. The software is being used mostly in a so called "black box" form with limited and very often insufficient description.

With programmes available to solve most any problem, it would seem that the job of the engineer - analyst has been greatly simplified.

Some suggest that the analysis process can be automated and codified to the point where the user merely follows a cook book procedure and needs to know little about the computer software he is using. Others suggest that programmes can be written with a sufficient number of internal checks and automated procedures so that it is impossible for the engineer to misuse the software. These may be desirable objectives, but they are not easily attained. There are numerous components, involved in aircraft design, there are many different computer programmes based on different concepts, and new software is being developed all the time. Each new configuration requires a new set of approximation, and each new computer programme treats the solution differently.

The computer has taken much of the drudgery out of generating numbers, and has provided the engineer with solution methods that are potentially more accurate, but it has not relieved him of making decisions concerning its use. In fact, it gives him more decisions to make which require a greater knowledge of the basic theories involved and how they are approximated by the various computer programmes. In order to make these decisions properly, he must have a detailed understanding of the software and how it functions. He must understand the physical theory, the numerical approximations, the solution methods, and be familiar with peculiarities of specific computer programmes.

Used properly, the computer can provide a much more accurate representation of the response of a complex structure to complicated loading systems. But used improperly, it allows the user to generate wrong answers faster and at greater expense.

If we are to utilize fully the current state-of-the-art, we must depend on engineering analyst to exercise engineering judgement. He is ultimately responsible for qualification of the solution method and accuracy of results. This requires a user who is not only competent in the fundamentals of engineering but also sophisticated in the ways of computer software.

There is nothing new about depending on the engineering judgement of the design analyst. Many handbooks provides us with many formulas, but the engineer is responsible for deciding which one is applicable and what parameters should be used to approximate the solution for a particular structure. He cannot use thin beam theory formulas for deep beams where shear deformation is important, or for beams where out-of-plane deflections may occur. He cannot use elasticity theory to calculate deflections or buckling loads where the resulting stresses are beyond the yield strength of the material.

What is new in the application of computer software is that a rather large body of specialized knowledge is required, which until recently was not included in any of the formal engineering curricula. The Professional Engineer examinations do not cover this material, nor should they, due to its specialized nature. But it is necessary to protect the aircraft industry and the public in general, from misuse of software simply due to ignorance of user, thus, it would seem that in addition to verification of computer programmes and qualification of solution methods, it is necessary to qualify the users of computer software. It can be achieved only through a formal certification or licensing programme.

For a certification programme to be successful there must be:

- a clearly defined need for certification
- a strong motivation on the part of those certifying and being certified
- a definable body of knowledge upon which certification can be based.

The need for certification has already been developed in terms of dependence on engineering judgement in an area which involves a large body of specialized knowledge. The responsibility for the proper use of sophisticated software falls upon a relatively few, and it is necessary to insure their competence.

Motivation should be considered in terms of professionalism and the harm that can be done to individuals and to society by users of computer software. Individual practitioners in other professions - for example in law and medicine - can do a great deal of harm, but usually only to one person at a time. Engineers can harm many more people with a single stroke, but the chances of this happening in pre-computer days was relatively slight because engineers tend to work in teams and their work is usually checked extensively. However, with the large-scale computer, voluminous results can be generated rapidly and the aura accorded their authenticity is unreal. Software users should be strongly motivated toward certification to prohibit unknowledgeable users from giving the profession a bad name by unintentional but harmful mistakes.

Managers should also be motivated toward certification since they are almost completely at the mercy of the software user-analysts. Simple cursory checks are no longer possible - the only assurance of accuracy of a complex analysis is by independent third party checking.

The many ways of certification of the users have been considered over the last 15-20 years. They can be divided into three options:

- a mandatory apprenticeship period (say 3 to 5 years) the way as it used to be in aircraft industry in 1950s
- a "Finite Element Analyst" voluntary certification with a recertification after 5 years
- a Finite Element Analyst certification for a specified code.

Although the issue of the quality of the software user is extremely important, it is a very controversial one, difficult to apply nationally and unlikely to occur in the near future. User education and training, however, are crucial, realizable goals and should be in every organisation budget.

REPLACEMENT OF STRUCTURAL TESTS BY A FINITE ELEMENT ANALYSIS

The structural tests (static, dynamic and fatigue) are becoming more and more costly and time consuming. With the increasing power of computers, decrease of the cost of analysis and with the improvement of software, there is a temptation to omit the experimental test completely. The pressure already exist, but it is a very risky decision and can be eventually applied only after a very careful consideration.

In the "old days", before the era of computers, the structural analysis of an aircraft was performed manually using slide rules and desk calculators. Each step of the analysis could be easily verified. The final verification of the safety of the structure was done by experimental test.

In the contemporary finite element computer analysis, the programme is used in so called "black box" form, without or with a very limited possibility of verification of the analysis.

With the use of computers the size of analysis has increased dramatically, from let us say about 5 critical load cases for a typical aircraft to over 100. A lot of output is produced but it does not mean that the results of the contemporary computer analysis are automatically much better. The critical part of analysis is the idealization of the structure, even the most sophisticated finite element analysis is only as good, as good is the model. For any idealization many details are neglected - details which may be critical for the safety of the structure.

As an example, a photo of the cracked spar is shown in Figure 28. In the finite element analysis the spar was modelled with beam elements and the holes were neglected. The calculated stresses were well within the limits. The holes increased locally the stresses causing the failure. It is a good example showing that the used idealization was not adequate. Theoretically, it is possible to model properly every hole and every local stress concentration region, but practically it is difficult to achieve.

Therefore the tests of the structure are necessary - it is difficult to replace them until the finite element model is able to predict correctly the local stresses in the whole structure.

The finite element analysis is a very powerful tool and when used by qualified users, it can give a lot of information about the strength and the behaviour of the structure. It can also be used for explanation of experimental tests. But when used carelessly, it may produce wrong results.

In the cases in which the strength of the structures is verified by the experimental test, the further modification of the structure can be certified on the basis of the finite element analysis only.

When tests are not possible, in the process of certification one has to take into account the reliability of the finite element programme, user qualification and his experience with the performed type of analysis as applied to the particular structure.

It is also desired to repeat the analysis with another idealization and with another method (eg. boundary element method, finite difference method). All regions where locally may exist stress concentration should be examined carefully.

In the case of a dynamic analysis, a verification of natural frequencies should be done carefully, because the computer may miss some, without any warning.

CONCLUSIONS

The accuracy of numerical analyses is dependent both on the set of assumptions required in order to perform the analysis and on the validity of the analysis method itself. The experimental test goes a long way in providing evidence to substantiate a numerical study and in a complementary manner the numerical work should provide an adequate explanation of the experimental test. If the experimental test is not possible, the substantiation of one analysis against another can only be sufficient if completely different analysis methods are used.

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TABLE 1

BOUNDARY CONDITIONS AND THEORETICAL SOLUTIONS FOR PATCH TESTS (Ref. 27)

(a) Membrane Plate Patch TestBoundary Conditions: $u = 10^{-3}(x + y/2)$

$$v = 10^{-3}(y + x/2)$$

Theoretical Solution:

$$\epsilon_x = \epsilon_y = \gamma = 10^{-3}; \sigma_x = \sigma_y = 1333.; \tau_{xy} = 400.$$

(b) Bending Plate Patch TestBoundary Conditions: $w = 10^{-3}(x^2 + xy + y^2)/2$

$$\theta_x = \partial w / \partial y = 10^{-3}(y + x/2)$$

$$\theta_y = -\partial w / \partial x = 10^{-3}(-x - y/2)$$

Theoretical Solution:

Bending moments per unit length:

$$m_x = m_y = 1.111 \times 10^{-7}; m_{xy} = 10^{-7}$$

Surface stresses:

$$\sigma_x = \sigma_y = \pm .667; \tau_{xy} = \pm .200$$

(c) Solid Patch TestBoundary Conditions: $u = 10^{-3}(2x + y + z)/2$

$$v = 10^{-3}(x + 2y + z)/2$$

$$w = 10^{-3}(x + y + 2z)/2$$

Theoretical Solution:

$$\epsilon_x = \epsilon_y = \epsilon_z = \gamma_{xy} = \gamma_{yz} = \gamma_{zx} = 10^{-3}$$

$$\sigma_x = \sigma_y = \sigma_z = 2000; \tau_{xy} = \tau_{yz} = \tau_{zx} = 400$$



Figure 28. Photograph of the cracked spar - finite element model neglected the holes and the performed analysis showed stresses within the acceptable range. After taking into account the holes the local stresses were higher than yield stress.

-20-

REPORT DOCUMENTATION PAGE			
1. Recipient's Reference	2. Originator's Reference	3. Further Reference	4. Security Classification of Document
	AGARD-R-748	ISBN 92-834-0423-2	UNCLASSIFIED
5. Originator	Advisory Group for Aerospace Research and Development North Atlantic Treaty Organization 7 rue Ancelle, 92200 Neuilly sur Seine, France		
6. Title	RELIABILITY STUDIES OF FINITE ELEMENT METHODS IN NORTH AMERICA		
7. Presented at	the 62nd Meeting of the Structures and Materials Panel of AGARD, Høvik (Oslo), Norway, 13-18 April 1986.		
8. Author(s)/Editor(s)	Jerzy J. Kacprzynski		9. Date July 1987
10. Author's/Editor's Address			11. Pages 26
12. Distribution Statement	This document is distributed in accordance with AGARD policies and regulations, which are outlined on the Outside Back Covers of all AGARD publications.		
13. Keywords/Descriptors	Numerical analysis Aviation safety Safety engineering Reliability Proving Checking		
14. Abstract	<p>This report discusses the reliability of finite element analysis techniques in place of structural test in the context of airworthiness assessment. Using an example as illustration, the paper shows that in order to obtain the meaningful results, the analysis must be performed with extreme care by an experienced analyst. The reliance on the finite element analysis alone may result in serious misdiagnosis. The verification and the certification of programmes and the certification of users are discussed. Some of the most important tests for validation of finite element programmes are presented.</p> <p><i>(Keywords: NATO; aviation safety; safety engineering; reliability; computerized simulation)</i></p>		

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